Three of the following questions will serve as problems on the final exam:

1. Formulate the definition of $\lim\limits_{n\to\infty} a_n$
2. Formulate the definition of $\lim\limits_{x\to a} f(x)$
3. Prove that if $\lim\limits_{n\to\infty} |a_n| = 0$, then $\lim\limits_{n\to\infty} a_n = 0$.
4. Formulate the squeeze theorem for sequences.
5. Formulate the monotonic sequence theorem.
6. Write the formula for the sum of geometric series.
7. Prove that if the series $\sum\limits_{n=1}^{\infty} a_n$ is convergent, then $\lim\limits_{n\to\infty} a_n = 0$.
8. Formulate the test for divergence.
9. Formulate the integral test.
10. Formulate the p-test.
11. Formulate the comparison test.
12. Formulate the limit comparison test.
13. Formulate the alternating series test.
14. Prove that if a series is absolutely convergent, then it is convergent.
15. Formulate the ratio test.
16. Write the Taylor formula.
17. Write the Maclaurin series for e^x.
18. Write the Maclaurin series for $\sin x$.
19. Write the Maclaurin series for $\cos x$.
20. Write the Maclaurin series for $\ln(1+x)$.

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21. Given vector $\vec{\mathbf{a}} = \langle a_1, a_2, a_3 \rangle$, what is its magnitude?

22. Let θ be the angle between vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$. What is $\vec{\mathbf{a}} \circ \vec{\mathbf{b}}$?

23. Let $\vec{\mathbf{a}} = \langle a_1, a_2, a_3 \rangle$, $\vec{\mathbf{b}} = \langle b_1, b_2, b_3 \rangle$. Write the formula for $\vec{\mathbf{a}} \circ \vec{\mathbf{b}}$ in terms of $a_1, a_2, a_3, b_1, b_2, b_3$.

24. Given vector $\vec{\mathbf{a}}$, find the unit vector $\vec{\mathbf{u}}$ having the same direction.

25. Write the formula for the scalar projection of $\vec{\mathbf{a}}$ onto $\vec{\mathbf{b}}$.

26. Write the formula for the vector projection of $\vec{\mathbf{a}}$ onto $\vec{\mathbf{b}}$.

27. Let θ be the angle between vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$. What is $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|$?

28. Let $\vec{\mathbf{a}} = \langle a_1, a_2, a_3 \rangle$, $\vec{\mathbf{b}} = \langle b_1, b_2, b_3 \rangle$. Write the formula for $\vec{\mathbf{a}} \times \vec{\mathbf{b}}$ in terms of $a_1, a_2, a_3, b_1, b_2, b_3$.

29. What is the scalar triple product of vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, and $\vec{\mathbf{c}}$?

30. Let $\vec{\mathbf{a}} = \langle a_1, a_2, a_3 \rangle$, $\vec{\mathbf{b}} = \langle b_1, b_2, b_3 \rangle$, $\vec{\mathbf{c}} = \langle c_1, c_2, c_3 \rangle$. Write the formula for the scalar triple product of these vectors.

31. Write the formula for the area of the parallelogram formed by vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$.

32. Write the formula for the volume of the parallelepiped formed by vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, $\vec{\mathbf{c}}$.

33. Write the equation of the line with directional vector $\vec{\mathbf{v}}$, going through point $P(x_0, y_0, z_0)$:
(a) in vector form (b) in parametric form (c) in symmetric form.

34. Write the equation of the plane with normal vector $\vec{\mathbf{n}}$ going through point $P(x_0, y_0, z_0)$:
(a) in vector form (b) in scalar form.

35. Write the equation of tangent line to the curve $\vec{\mathbf{r}}(t)$ at point $P(x_0, y_0, z_0)$.

36. Write the formula for the length of curve $\vec{\mathbf{r}}(t)$ if $a \leq t \leq b$.

37. Let $\vec{\mathbf{r}}(t)$ be the position vector of a particle. Write the formula for its velocity $\vec{\mathbf{v}}(t)$ and acceleration $\vec{\mathbf{a}}(t)$.

38. Let $\vec{\mathbf{v}}(t)$ be the velocity of a particle. Write the formula for its position vector $\vec{\mathbf{r}}(t)$ if at time t_0 the particle was located at point P with radius-vector $\vec{\mathbf{r}}_0$.

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39. What is the definition of $f_x(x, y)$?

40. Formulate the Clairaut Theorem (about mixed derivatives).

41. Write the definition of a differentiable function of two variables.

42. What is the differential of function $f(x, y)$?

43. Given surface $\vec{\mathbf{r}}(u, v)$, what is the normal vector to the tangent plane?

44. What is the equation of the tangent plane to the surface $z = f(x, y)$ at the point $P(x_0, y_0, z_0)$?

45. Given surface $z = f(x, y)$, what is the normal vector to the tangent plane?

46. Write the formula for $\dfrac{df}{dt}$ (Chain rule) for function $f(x, y)$ if $x = x(t)$, $y = y(t)$.

47. Write the formula for $\dfrac{\partial f}{\partial u}$ (Chain rule) for function $f(x, y)$ if $x = x(u, v)$, $y = y(u, v)$.

48. Write the definition of the derivative in the direction of unit vector $\vec{\mathbf{u}} = \langle a, b \rangle$ and the formula connecting the directional derivative and partial derivatives.

49. Write the definition of the gradient vector and the formula connecting the directional derivative and gradient.

50. Formulate the theorem on maximizing the directional derivative.

51. Given surface $F(x, y, z) = k$, write the formula for the tangent plane at point (x_0, y_0, z_0).

52. Given surface $F(x, y, z) = k$, what is the normal vector to the tangent plane?

53. Given function $f(x, y)$, prove that the gradient vector is perpendicular to level curves of f.

54. Formulate the second derivative test for extremum values of function $f(x, y)$.

55. Write the system of equations for the search for extremum values of function $f(x, y, z)$ under constraints $g(x, y, z) = k$ (Lagrange multipliers formula).

56. Write the definition of double integral of function $f(x, y)$ over rectangle R.

57. What is the formula for the volume V of the solid that lies above the region R on the xy coordinate plane and below the surface $z = f(x, y)$ if $f \geq 0$?

58. What is the average value of function $f(x, y)$ defined on a domain R?

59. Formulate the Fubini theorem on rectangle $R = \{(x, y)|\ a \leq x \leq b,\ c \leq y \leq d\}$.

60. Express the Cartesian coordinates x and y in terms of polar coordinates r and θ.

61. Express polar coordinates r and θ in terms of Cartesian coordinates x and y.

62. What is the expression for elementary area dA in polar coordinates?

63. What is the formula for the coordinates of the mass center of a thin plate with plane density $\rho(x, y)$?

64. Let X and Y be random variables and $f(x, y)$ be their joint density function. Write the formula for the probability the (X, Y) lies in a region D.

65. Show that $\int\limits_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$.

66. Show that $\int\limits_{-\infty}^{+\infty} e^{-x^2/2} dx = \sqrt{2\pi}$.